

10c

$$F(\vec{x}) = \begin{pmatrix} y+z+1 \\ y^2-z-4 \end{pmatrix} \stackrel{!}{=} 0$$

$$\vec{x}_i = \begin{pmatrix} y_i \\ z_i \end{pmatrix}$$

$$J_F = \begin{pmatrix} 1 & 1 \\ 2y & -1 \end{pmatrix}$$

Iteration:

1) Löse nach  $\vec{a}$ :

$$J_F(\vec{x}_i) \cdot \vec{a} = -F(\vec{x}_i)$$

$$2) \vec{x}_{i+1} = \vec{x}_i + \vec{a}$$

$$\vec{x}_0 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} a_y \\ a_z \end{pmatrix} = - \begin{pmatrix} -1+1+1 \\ 1-1-4 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$a_z = 2$$

$$a_y = -1 - 2 = -3$$

$$\vec{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\vec{x}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

```
In[19]:= f[x_] := Module[{y = x[[1]], z = x[[2]]},  
      {y + z + 1,  
      y^2 - z - 4} /; VectorQ[x]
```

```
In[20]:= f[{1.3028, -2.3027}]
```

```
Out[20]= {0.0001, -0.00001216}
```

```
In[21]:= f[{-2.3028, 1.3028}]
```

```
Out[21]= {0., 0.00008784}
```

```
In[5]:= JacobianMatrix[f_List?VectorQ, x_List] := Outer[D, f, x] /; Equal@@(Dimensions/@{f, x})
```

```
In[22]:= J[x_] := (JacobianMatrix[f[{y, z}], {y, z}] /. y -> x[[1]]) /. z -> x[[2]]
```

```
In[23]:= x0 = {-1, 1};
```

Iteration:

```
In[35]:= a = LinearSolve[J[x0], -f[x0]]
```

```
      x1 = x0 + a
```

```
Out[35]= {-3, 2}
```

```
Out[36]= {-4, 3}
```

```
In[37]:= a = LinearSolve[J[x1], -f[x1]]
```

```
      x2 = x1 + a
```

```
Out[37]= {9/7, -9/7}
```

```
Out[38]= {-19/7, 12/7}
```

```
In[39]:= a = LinearSolve[J[x2], -f[x2]]
```

```
      x3 = x2 + a
```

```
Out[39]= {81/217, -81/217}
```

```
Out[40]= {-508/217, 291/217}
```

```
In[41]:= N[%]
```

```
Out[41]= {-2.34101, 1.34101}
```

```
In[42]:= a = LinearSolve[J[x3], -f[x3]]
```

```
      x4 = x3 + a
```

```
Out[42]= {6561/173383, -6561/173383}
```

```
Out[43]= {-399331/173383, 225948/173383}
```

```
In[44]:= N[%]
```

```
Out[44]= {-2.30317, 1.30317}
```

```
In[46]:= f[x4] // N
```

```
Out[46]= {0., 0.00143195}
```

```
In[47]:= a = LinearSolve[J[x4], -f[x4]]  
x5 = x4 + a
```

```
Out[47]= {  
   $\frac{43\,046\,721}{108\,412\,748\,857}$ ,  $-\frac{43\,046\,721}{108\,412\,748\,857}$   
}
```

```
Out[48]= {  
   $-\frac{249\,650\,241\,628}{108\,412\,748\,857}$ ,  $\frac{141\,237\,492\,771}{108\,412\,748\,857}$   
}
```

```
In[50]:= f[x5] // N
```

```
Out[50]= {0.,  $1.57659 \times 10^{-7}$ }
```