

Artificial Intelligence 2 – SS 2020

Assignment 2: Conditional Probabilities

– Given May 1., Due May 10. –

Hint: Exercises need to be handed in via StudOn at 23:59 on the day they are due or earlier. Please use only the exercise group of your tutor to hand in your work.

If any concepts here seem unfamiliar to you or you have no idea how to proceed, consult the lecture materials, ask a fellow student, your tutor, or on the Forum.

If a problem asks for code, comment it or make it otherwise self-explanatory. You do not need to write a lot, but it should be enough to convince your tutor that you understand what the code does. We may deduct up to 30% for uncommented and unclear code, but would prefer not to.

Problems with no points (0pt) will not be graded, but might appear on the exam in a similar form. For these, we will provide a reference solution after the submission deadline. If you find the reference solution unclear, ask about it on the forum or in a tutorial.

Problem 2.1 (Bayesian Rules)

Name and write down the first four basic rules in Bayesian inference from the first two weeks of lectures. 0pt

Problem 2.2 (AFT Tests)

Trisomy 21 (*Down syndrome*) is a genetic anomaly that can be diagnosed during pregnancy using an amniotic fluid test. 40pt

The probability of a foetus having Down syndrome is strongly correlated with the age of the pregnant parent. We will only consider the following two age groups.

1. For 25 year olds the probability is one in 1250,
2. for 43 year old parents it increases to one in fifty.

However, diagnostic tests are never perfect. We distinguish two kinds of errors:

3. Type I Error (False Positive): The test result is positive even though the child is healthy.
4. Type II Error (False Negative): The test result is negative even though the child has trisomy 21.

The probabilities of Type I and Type II Errors are both merely 1% for amniotic fluid tests for Down syndrome.

10 pt

1. Express the four items above in the form of conditional probabilities. Use the random variable F with Domain $\{Age_{25}, Age_{43}\}$ for the age of the pregnant person and the boolean random variables Pos and $Down$ for the propositions “*The amniotic fluid test is positive*” and “*The child has Down syndrome*” respectively.

30 pt

2. Assume that we have a 25 year old pregnant person. Using Bayes’ theorem, express and compute the probability that their child has Down syndrome, given that the amniotic fluid test is positive. What can we conclude from the result?

Problem 2.3 (Chained Production Elements)

An apparatus consists of six element A, B, C, D, E, F . The apparatus works if and only if at least A and B are operational, C and D are operational, or E and F are operational.

40pt

20 pt

1. Assume the probabilities $P(X)$, that element X breaks down, are all stochastically independent, with $P(A) = 5\%$, $P(B) = 10\%$, $P(C) = 15\%$, etc. What is the probability the apparatus works?

20 pt

2. Consider a different scenario, in which the elements A and C , D and F and B and E are pairwise linked; such that if either of them breaks, then the linked element is not operational either. What is the probability that the apparatus works now?

(**Note** that we deliberately differentiate between *not being operational* and *being broken*! If an element breaks, it is not operational; if an element is not operational, either it or the linked element broke)