

Luncheon Adventures

A bonus exercise for Hidden Markov Models

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Problem X.1 (More HMMs):

Say you join a work group at your university. Social convention dictates that you go to lunch with them at the university cafeteria every day (that is, Monday to Friday). Most people in your new circle are omnivores, but your co-worker Max is a vegan.

That complicates things because the cafeteria doesn't always offer a vegan lunch menu option. They do however, always have a salad bar that can be used to assemble a vegan salad.

You heard Max say at some point, that if there *is* a vegan lunch option, they only pick the salad one out of six times because the cooked food is usually more varied and tastier.

Max also likes to cook and sometimes (say, 10% of the time) prepares a meal ahead of time at home and eats that at lunch, regardless of whether there was a vegan offer that day or not.

If there is no vegan option and no meal prepared ahead of time, they always pick the salad (Max never goes hungry).

In an amazing strike of coincidence, you also overhear some cafeteria workers on the bus home some day. From them, you learn that when they had one day without a vegan option, they won't have one the next day in nine out of ten cases. But when they finally get around to offering a vegan option, they feel good about themselves and are 30% likely to offer one the next day as well.

Note that you cannot directly observe the cafeteria's menu offers, since you always bring food from home and care little for surroundings when you are hungry.

1. It is Thursday afternoon and you notice that tomorrow will be International Prank Your Co-workers Day. You recently acquired some edible fake worms and cockroaches and for some reason think it would be *hilarious* instead of annoying to smuggle them into Max's lunch. But it will only really work if they have a salad tomorrow.

You know there was a vegan option on Monday and that Max has eaten salad on all other days this week so far. What is the probability that Max will also eat salad tomorrow?

2. One (different) Friday, while munching on a vegan burrito, Max remarks that their birthday is coming up and that they don't want to be stuck with salad on their birthday but also don't want to go through the hassle of cooking ahead. They wonder out loud what the probability might be that the cafeteria will have a vegan offer next Tuesday. Calculate and state that probability.
3. Exactly two weeks later, Max is in a bad mood and complains about how there was no vegan option *again* today and that there probably won't be one for the whole of next week, either. What are the chances that Max was too pessimistic and there will be *at least one* vegan lunch option next week?