
Aufgabe 1

$$T(n) \leq \begin{cases} a & \text{if } n = 1, \\ C\sqrt{n} + T(\lceil \frac{n}{2} \rceil) & \text{if } n > 1. \end{cases}$$

Case $n = 2^k$:

$$T(n) = T(2^k) \leq C\sqrt{2^k} + T(2^{k-1}) = C\sqrt{2^k} + (C\sqrt{2^{k-1}} + T(2^{k-2})) = C(\sqrt{2^k} + \sqrt{2^{k-1}} + \sqrt{2^{k-2}} \dots \sqrt{2}) + T(2^0) = C \sum_{i=0}^{k-1} \sqrt{2^{k-i}} + a = a + C\sqrt{2^k} \sum_{i=0}^{k-1} \sqrt{2^{-i}} \stackrel{W.A.}{=} a + C\sqrt{2^k} \cdot \frac{2(\sqrt{2^k-1})}{\sqrt{2}-2}$$

(According to Wolfram Alpha, can be proofed easily by Induction over k)

$$\Rightarrow T(n) \leq 2C \frac{1-\sqrt{n}}{\sqrt{2}-2} + a \text{ if } n = 2^k$$