

1 Deduction

1.1 Propositional Natural Deduction Calculus \mathcal{ND}^0

$$\begin{array}{c}
 \frac{A \quad B}{A \wedge B} I_{\wedge} \qquad \frac{A \wedge B}{A} E_{\wedge} \qquad \frac{A \wedge B}{B} E_{\wedge} \\
 \\
 \frac{\frac{[A]^1}{B} I_{\Rightarrow}^1}{A \Rightarrow B} I_{\Rightarrow}^1 \qquad \frac{A \Rightarrow B \quad A}{B} I_{\Rightarrow} \\
 \\
 \frac{A}{A \vee B} I_{\vee} \qquad \frac{B}{A \vee B} I_{\vee} \qquad \frac{A \vee B \quad [A]^1 \vdash C \quad [B]^1 \vdash C}{C} E_{\vee}^1 \qquad \frac{}{A \vee \neg A} TND \\
 \\
 \frac{[A]^1 \vdash \perp}{\neg A} I_{\neg}^1 \qquad \frac{\neg \neg A}{A} E_{\neg} \\
 \\
 \frac{\neg A \quad A}{\perp} I_{\perp} \qquad \frac{\perp}{A} E_{\perp}
 \end{array}$$

1.2 First-Order Natural Deduction \mathcal{ND}^1

$$\begin{array}{c}
 \frac{A}{\forall X.A} I_{\forall} \qquad \frac{\forall X.A}{[B/X](A)} E_{\forall} \\
 \\
 \frac{[B/X](A)}{\exists X.A} I_{\exists} \qquad \frac{\exists X.A \quad [[c/X](A)]^1 \vdash C}{C} E_{\exists} \\
 \\
 \frac{}{A = A} I_{=} \qquad \frac{A = B \quad C[A]_p}{[B/p]C} E_{=} \\
 \\
 \frac{}{A \Leftrightarrow A} I_{=} \qquad \frac{A \Leftrightarrow B \quad C[A]_p}{[B/p]C} E_{=}
 \end{array}$$

2 Automated Theorem Proving

2.1 Analytical Tableau \mathcal{T}^0

$$\begin{array}{c}
 \frac{A \wedge B^T}{A^T} \\
 B^T \\
 \frac{A \vee B^T}{A^T \mid B^T} \\
 \frac{\neg A^T}{A^F} \\
 \\
 \frac{A^\alpha \quad A^\beta \quad \alpha \neq \beta}{\perp} \\
 \\
 \frac{A \Rightarrow B^T}{A^F \mid B^T} \quad \frac{A \Rightarrow B^F}{A^F \mid B^T} \quad \frac{A^T}{A \Rightarrow B^T} \\
 \frac{A \Leftrightarrow B^T}{A^T \mid A^F \mid B^T} \quad \frac{A \Leftrightarrow B^F}{A^T \mid A^F \mid B^T}
 \end{array}$$

2.2 First-Order Tableaux \mathcal{T}^1

$$\begin{array}{c}
 \frac{\forall X.A^T \quad C \in \text{cwff}_t(\Sigma_t)}{[C/X](A)^T} \quad \frac{\forall X.A^F \quad c \in (\Sigma_o^{\text{sk}} \setminus \mathcal{H})}{[c/X](A)^F} \\
 \\
 \frac{\forall X.A^T \quad Y_{\text{new}}}{[Y/X](A)^T} \quad \frac{\forall X.A^F \quad \text{free}(\forall X.A) = \{X^1, \dots, X^k\} \quad f \in \Sigma_k^{\text{sk}}}{[f(X^1, \dots, X^k)/X](A)^F} \\
 \\
 \frac{A^\alpha \quad B^\beta \quad \alpha \neq \beta \quad \sigma(A) = \sigma(B)}{\perp : \sigma} \\
 \\
 \frac{\exists X.A^F}{\forall X.\neg A^T} \quad \frac{\exists X.A^T}{\forall X.\neg A^F}
 \end{array}$$

3 Resolution

3.1 Resolution for Propositional Logic

3.1.1 Clause Normal Form Transformation CNF^0

$$\frac{C \vee (A \vee B)^T}{C \vee A^T \vee B^T} \qquad \frac{C \vee (A \vee B)^F}{C \vee A^F; C \vee B^F}$$

$$\frac{C \vee \neg A^T}{C \vee A^F} \qquad \frac{C \vee \neg A^F}{C \vee A^T}$$

$$\frac{C \vee (A \Rightarrow B)^T}{C \vee A^F \vee B^T} \qquad \frac{C \vee (A \Rightarrow B)^F}{C \vee A^T; C \vee B^F}$$

$$\frac{C \vee (A \wedge B)^T}{C \vee A^T; C \vee B^F} \qquad \frac{C \vee (A \wedge B)^F}{C \vee A^F \vee B^F}$$

3.1.2 Resolution Calculus \mathcal{R}^0

$$\frac{P^T \vee A \quad P^F \vee B}{A \vee B}$$

3.2 First-Order Resolution

3.2.1 Conjunctive Normal Form Calculus CNF^1

$$\frac{\forall X. A^T \vee C \quad Z \notin (\text{free}(A) \cup \text{free}(C))}{[Z/X](A)^T \vee C} \qquad \frac{\forall X. A^F \vee C \quad \{X^1, \dots, X^k\} = \text{free}(\forall X. A)}{[f_n^k(X^1, \dots, X^k)/X](A)^F \vee C}$$

3.2.2 First-Order Resolution Calculus \mathcal{R}^1

$$\frac{A^T \vee C \quad B^F \vee D \quad \sigma = \text{mgu}(A, B)}{\sigma(C) \vee \sigma(D)} \qquad \frac{A^\alpha \vee B^\alpha \vee C \quad \sigma = \text{mgu}(A, B)}{\sigma(A) \vee \sigma(C)}$$