1 Deduction

1.1 Propositional Natural Deduction Calculus $\mathcal{ND}^0$

$$\frac{A \quad B}{A \wedge B}\, I_\wedge \qquad \frac{A \wedge B}{A}\, E_\wedge \qquad \frac{A \wedge B}{B}\, E_\wedge$$

$$\frac{\frac{[A]^1}{B}}{A \Rightarrow B}\, I^1_\Rightarrow \qquad \frac{A \Rightarrow B \quad A}{B}\, I_\Rightarrow$$

$$\frac{A}{A \vee B}\, I_\vee \qquad \frac{B}{A \vee B}\, I_\vee \qquad \frac{A \vee B \quad [A]^1 \vdash C \quad [B]^1 \vdash C}{C}\, E^1_\vee \qquad \frac{}{A \vee \neg A}\, TND$$

$$\frac{[A]^1 \vdash \bot}{\neg A}\, I^1_\neg \qquad \frac{\neg\neg A}{A}\, E_\neg$$

$$\frac{\neg A \quad A}{\bot}\, I_\bot \qquad \frac{\bot}{A}\, E_\bot$$

1.2 First-Order Natural Deduction $\mathcal{ND}^1$

$$\frac{A}{\forall X.A}\, I_\forall \qquad \frac{\forall X.A}{[B/X]\,(A)}\, E_\forall$$

$$\frac{[B/X]\,(A)}{\exists X.A}\, I_\exists \qquad \frac{\exists X.A \quad [[c/X]\,(A)]^1 \vdash C}{C}\, E_\exists$$

$$\frac{}{A = A}\, I_= \qquad \frac{A = B \quad C[A]_p}{[B/p]\,C}\, E_=$$

$$\frac{}{A \Leftrightarrow A}\, I_= \qquad \frac{A \Leftrightarrow B \quad C[A]_p}{[B/p]\,C}\, E_=$$

2 Automated Theorem Proving

2.1 Analytical Tableau $\mathcal{T}^0$

$$\frac{A \wedge B^T}{\begin{array}{c}A^T\\B^T\end{array}} \qquad \frac{A \wedge B^F}{A^F \mid B^F}$$

$$\frac{A \vee B^T}{A^T \mid B^T} \qquad \frac{A \vee B^F}{\begin{array}{c}A^F\\B^F\end{array}}$$

$$\frac{\neg A^T}{A^F} \qquad \frac{\neg A^F}{A^T}$$

$$\frac{\begin{array}{c}A^\alpha\\A^\beta\end{array} \quad \alpha \neq \beta}{\bot}$$

$$\frac{A \Rightarrow B^T}{A^F \mid B^T} \qquad \frac{A \Rightarrow B^F}{\begin{array}{c}A^T\\B^F\end{array}} \qquad \frac{\begin{array}{c}A^T\\A \Rightarrow B^T\end{array}}{B^T}$$

$$\frac{A \Leftrightarrow B^T}{\begin{array}{c}A^T\\B^T\end{array} \mid \begin{array}{c}A^F\\B^F\end{array}} \qquad \frac{A \Leftrightarrow B^F}{\begin{array}{c}A^T\\B^F\end{array} \mid \begin{array}{c}A^F\\B^T\end{array}}$$

2.2 First-Order Tableaux $\mathcal{T}^1$

$$\frac{\forall X.A^T \quad C \in \mathrm{cwff}_\iota(\Sigma_\iota)}{[C/X]\,(A)^T} \qquad \frac{\forall X.A^F \quad c \in (\Sigma_o^{\mathrm{sk}} \setminus \mathcal{H})}{[c/X]\,(A)^F}$$

$$\frac{\forall X.A^T \quad Y \text{ new}}{[Y/X]\,(A)^T} \qquad \frac{\forall X.A^F \quad \mathrm{free}(\forall X.A) = \{X^1, \ldots, X^k\} \quad f \in \Sigma_k^{\mathrm{sk}}}{[f(X^1, \ldots, X^k)/X]\,(A)^F}$$

$$\frac{\begin{array}{c}A^\alpha\\B^\beta\end{array} \quad \alpha \neq \beta \quad \sigma(A) = \sigma(B)}{\bot : \sigma}$$

$$\frac{\exists X.A^F}{\forall X.\neg A^T} \qquad \frac{\exists X.A^T}{\forall X.\neg A^F}$$

3 Resolution

3.1 Resolution for Propositional Logic

3.1.1 Clause Normal Form Transformation $\boldsymbol{CNF}^0$

$$\frac{C \vee (A \vee B)^T}{C \vee A^T \vee B^T} \qquad \frac{C \vee (A \vee B)^F}{C \vee A^F ; C \vee B^F}$$

$$\frac{C \vee \neg A^T}{C \vee A^F} \qquad \frac{C \vee \neg A^F}{C \vee A^T}$$

$$\frac{C \vee (A \Rightarrow B)^T}{C \vee A^F \vee B^T} \qquad \frac{C \vee (A \Rightarrow B)^F}{C \vee A^T ; C \vee B^F}$$

$$\frac{C \vee (A \wedge B)^T}{C \vee A^T ; C \vee B^F} \qquad \frac{C \vee (A \wedge B)^F}{C \vee A^F \vee B^F}$$

3.1.2 Resolution Calculus $\mathcal{R}^0$

$$\frac{P^T \vee A \quad P^F \vee B}{A \vee B}$$

3.2 First-Order Resolution

3.2.1 Conjunctive Normal Form Calculus $\boldsymbol{CNF}^1$

$$\frac{\forall X.A^T \vee C \quad Z \notin (\mathrm{free}(A) \cup \mathrm{free}(C))}{[Z/X]\,(A)^T \vee C} \qquad \frac{\forall X.A^F \vee C \quad \{X^1, \ldots, X^k\} = \mathrm{free}(\forall X.A)}{[f_n^k(X^1, \ldots, X^k)/X]\,(A)^F \vee C}$$

3.2.2 First-Order Resolution Calculus $\mathcal{R}^1$

$$\frac{A^T \vee C \quad B^F \vee D \quad \sigma = \mathrm{mgu}(A, B)}{\sigma(C) \vee \sigma(D)} \qquad \frac{A^\alpha \vee B^\alpha \vee C \quad \sigma = \mathrm{mgu}(A, B)}{\sigma(A) \vee \sigma(C)}$$