Optimization for Engineers

Final Exam
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Dr. Johannes Hild
Department Mathematatik
Universität Erlangen-Nürnberg

Assignment 1: Karush-Kuhn-Tucker

Consider the equality constrained problem

$$\text{minimize} \quad f(x) \quad \text{s.t.} \quad x \in \Omega := \{x \in \mathbb{R}^3 : h_1(x) = h_2(x) = 0\}$$

with $x = (u, v, w)^\top$ and $f(x) = \frac{4}{3}u^3 - v - 2w$ and $h_1(x) = u + v - 1$ and $h_2(x) = w - u$.

a) (2 pts) Show that the (LICQ) is satisfied at any point $x \in \Omega$.

b) (6 pts) Formulate the Lagrangian function $L(x, \lambda_1, \lambda_2)$ and find all points $(x^*, \lambda_1^*, \lambda_2^*)$ solving the (KKT) conditions for this problem.

c) (2 pts) Compute the Hessian $\nabla^2 f(x)$ and decide, which kind of definiteness holds for the Hessian at the (KKT)-points from b).

Assignment 2: Levenberg Marquardt Step

Consider a least squares problem

$$\text{minimize} \quad f(u,v) := \frac{1}{2}R(u,v)^\top R(u,v), \qquad \text{s.t.} \quad x = (u,v)^\top \in \mathbb{R}^2$$

with error vector $R(u,v) = (v - 3, uv - 3, u^2v - 3)^\top$ and $x_0 := (1,0)^\top$.

a) (6 pts) Find d_α satisfying $\left(J(x_0)^\top J(x_0) + \alpha E\right) d_\alpha = -\nabla f(x_0)$ and compute $x_\alpha = x_0 + d_\alpha$ in dependence of $\alpha > 0$. E is the unit matrix and $J(u,v)$ is the Jacobian of $R(u,v)$.

b) (2 pts) Compute $\lim_{\alpha \to 0} R(x_\alpha)$ and conclude the (GMP) x_* of f on $\mathbb{R}^2$.

c) (2 pts) Prove in general: If a vector set $\{p_i\}_{i=1}^n$ is both A-conjugate and B-conjugate, it is also $(A+B)$-conjugate.

Assignment 3: Projected Steepest Descent / Exact Line Search

Consider the problem

$$\text{minimize} \quad f(u,v) := u^3 + \frac{2}{v} + v^2, \qquad \text{s.t.} \quad x = (u,v)^\top \in \Omega_\square := [\frac{1}{2}, 3]^2$$

and the projection into box constraints $P : \mathbb{R}^2 \to \Omega_\square$ and $x_0 := (1,1)^\top$.

a) (6 pts) Compute $x_1 := P(x_0 + t_0 d_0)$ with d_0 is the steepest descent. t_0 is resulting from exact line search with respect the **unconstrained** problem: minimize $f(u,v)$ s.t. $x \in \mathbb{R}^2$.

b) (3 pts) Use second order optimality conditions to decide if $x_* = (\frac{1}{2}, 1)^\top$ is a nondegenerate (LMP).

c) (1 pts) Consider the general convex line search problem minimize $\phi(t) := F(x_k + td_k)$, s.t. $t \in [0,1]$. Name an algorithm that can be used to find a step size t_k that is reliably close to the (GMP) t_* of ϕ on $[0,1]$.

Assignment 4: Linear Programming

Consider the linear program in standard form

$$\begin{aligned} &\text{minimize} \quad f(x) := c^\top x \\ \text{s.t.} \quad &x \in \Omega := \{x \in \mathbb{R}^n : Ax = b \quad \text{and} \quad x_i \geq 0 \quad \text{for} \quad i = 1, \dots, n\} \end{aligned}$$

with data

$$A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \qquad c = \begin{pmatrix} 3 & 2 & 1 \end{pmatrix}^\top$$

a) (1 pts) Formulate the *Phase I*-Data $\tilde{A}$, $\tilde{b}$, $\tilde{c}$ for this problem.

b) (2 pts) Show that $x_0 = (\frac{1}{3}, \frac{4}{3}, 0)^\top$ is a (BFP) for Phase II and state the basis index set $\mathcal{B}_0$.

c) (6 pts) Execute *Phase II* with x_0 to solve the linear program.

d) (1 pts) State a reason, why *conjugate gradient* cannot be used to solve the linear equation systems occurring in simplex steps in general.