## 1. Introduction

- Explain all steps of the visualization pipeline!

1. Simulation
2. Raw Data
3. Filtering (Smoothing, missing values, select attributes..)
4. Extraction (Isosurfaces, fibersurfaces, extremal points, ridge lines..)
5. Mapping (Transfer function, visual cues, transformation, layouting...)
6. Synthesis (Ray marching, ray tracing, rasterization..)
7. Image, Human (-> interactive exploration, parameter adjustment)

- Why do we perform data abstraction?

To acquire application-independent language for the description of data to ease scientific visualization for each kind of problem/field

- Name and explain data types!

1. Attribute = property that can be measured or logged
2. Item = individual discrete entity
3. Link = relationship between items, typically within a network
4. Position $=$ spatial and/or temporal location
5. Grid = defines connectivity of positions for interpolation

- What is the difference between a link and a grid?

Link shows relationships (parent-child, tree..) -> spatial data
Grid is telling how to connect individual measurements to define -> abstract data

- Given a visualization, identify the items/links/attributes or positions/grid/ attributes, respectively!
- Which different types of attributes do exist?

1. Nominal/categorical (no order)
2. Ordinal (ordered: differences between values cannot be quantified, e.g. S, M, L, XL)
3. Quantitative (ordered: measured values represent numbers, distance can be measured), e.g. Time

- Which types of orderings do we distinguish?

1. Sequential (from min to max)
2. Diverging (there is a meaningful mid point)
3. Cyclic (data wraps around back to beginning)

- Categorize a given attribute!


## 2. Perception and Mapping

- Which types of marks exist?

Mark = basic graphical element in an image that represents items or links

1. Marks as item/node (points: 0d, lines: 1d, areas: 3d)
2. Marks as link (connection mark: pairwise relationship between two items;
containment mark: hierarchical relationship between areas)

- Which types of channels exist?

Channel = appearance of mark

1. Identity channels: what or where is something (categorical data)
2. Magnitude channels: how much of something is there (ordered data)

- Explain expressiveness and effectiveness!

Expressiveness: Visual encoding should express all of, and only!, the information in the attributes of the dataset. (-> Ordered data must show order in a way that the human perception system intrinsically senses as ordered)
Effectiveness: The importance of the attribute should match the salience of the channel, i.e. its noticeability. (-> the most important attribute should be encoded with the most effective channel in order to be most noticeable)

- Order encodings by their effectiveness!
$\Theta$ Magnitude Channels: Ordered Attributes
Position on common scale
Position on unaligned scale
Length (1D size)
Colt/angle
Color luminance (2D size)
Color saturation
Curvature
Volume (3D size)

Identity Channels: Categorical Attributes


## Ranking of Channel Effectivess

- How are encodings perceptually linearized?

Sublinear (doubling x -> smaller difference), linear (length), superlinear


- What is preattentive vision?

Rapid parallel processing at the retina to extract low-level properties, bottom up processing without special attention

- Name six differences that are perceived preattentively!

Addition, Gray/value, shape, orientation, curved/straight, shape

Addition


Orientation


Gray/value


Curved/straight



Shape


- Name two differences that require attention!

Parallelism, juncture

## Parallelism



Juncture


- Name channels that are appropriate to encode nominal/ordinal/quantitative data!
see Image
- How can groupings be encoded?
- What does Weber's law say?

Human perception is based on relative judgements, not absolute judgements

- Which biases exist in human perception?
- Explain the components of the HSL color space!

Hue: identity channel
Saturation: magnitude channel
Ligntness: magnitude channel

- Name guidelines for use of color!

Luminance in non-contiguous regions
Saturation: in non-contiguous regions
Hue: in contiguous regions, second most-effective channel for categorical data, third most for encoding groupings, maximize perceptual distance between colors Transparency: interferes with the other 3 , can be used in conjunction with hue

## 3. Scalar and Vector Fields

- How is the gradient of a scalar field calculated and what does it mean?

2D: $f(x, y), 3 D: g(x, y, z)$
$\operatorname{grad} \mathrm{f}=\binom{f^{\prime}(x)}{f^{\prime}(y)}$
grad $f$ is vector field, points in direction of maximal rate of increase of $f$
through every point where grad f != 0 passes an isocontour c

- Which vector field could be used to trace isolines?

The Co-gradient (points in direction of isolines of f)
co-grad $=\binom{-f^{\prime}(y)}{f^{\prime}(x)}$

- What are the vertices of a Morse-Smale cell?

Morse-Smale cell: each region is a quadrangle with vertices (Eckpunkte) minimum, saddle, maximum, saddle in this order around the region

- How can ridges in 2D and 3D scalar fields be defined?

Separating lines of Morse-Smale-Complex (needs only first order derivatives)
Ridge is connected to sink/maximum

- How can minima/maxima of scalar fields be distinguished?

Calculate gradient and set it to $0->\operatorname{grad} f(x)==0$
Set Nullstellen to Hesse Matrix, then calculate Eigenvalues E
Hesse Matrix $=\left(\begin{array}{ll}f_{x x} & f_{x y} \\ f_{y x} & f_{y y}\end{array}\right)$
Minimum: E1, E2 > 0
Maximum: E1, E2 < 0
Saddle: E1 < $0<$ E2

- How can sinks/saddles/sources in vector fields be distinguished?

- Can saddles in 2D/3D vector fields be swirling? Why?

No because swirling behavior requires the imaginary parts of the Eigenvalues to be unequal to 0 , but a saddle is defined by $\mathrm{Im}=0$

- Be ready to extract and classify critical points in analytic vector fields! (see exercises)
- What is the meaning of the Poincaré index?
= number of counterclockwise rotations of the vector
Index is correlating with the type of critical point in the region (saddle = -1, source/ sink/center = 1)
- What is the difference between stream lines, path lines, streak lines and time lines?
Stream lines: path of a massless particle in a flow described by vector field $v$ (do not intersect each other, only one stream line per point in v)
Path lines:
Streak lines:
Time lines:
- Be ready to compute Jacobian, divergence, curl and Laplace of a given scalar/vector field! (see exercises)


## 4. Data Structures

- What types of grid structures exist?

1. Cartesian (axis-aligned lines, constant distance, $x=y$ )
2. Regular (axis-aligned, constant distance, but may vary across $x$ and $y$ )
3. Rectilinear (axis-aligned, variable distance within and across dimensions)
4. Curvilinear (points located along parametric curves)
5. Unstructured (points arbitrarily distributed in space)

- Why are unstructured grids preferred in practice?

Because you need high resolution, can present more detail with them, better for simulation

- Given a set of scattered data points. How can their attributes be interpolated efficiently?
By applying scattered data triangulation
- How is triangle quality assessed?
- Given a linear vector field in barycentric coordinates. How are the vector field and its derivatives converted into Cartesian coordinates? u * P0 + v * P1 + w * P2
- Given some triangle (a, b, c), sketch the barycentric coordinates (u,v,w)=(1/2, $1 / 4,1 / 4$ ) and (u,v,w) $=(0,1 / 3,2 / 3)$
- Why is the update order in data transformation pipelines relevant?

To avoid redundant computations and improve performance

- Name common data formats!
.vti, .vtu, .vtp, .vtk, .nc


## 5. Indirect Volume Visualization

- Explain and apply the marching squares algorithm to a random $3 \times 3$ scalar grid for a given isovalue IV.
Algorithm:

1. For each edge: look if its value is greater or smaller than the given is-value, lines are only inserted for edges with ambiguous isovalues (one smaller, one bigger)
2. Find the point on each edge with an is-value of 0 :
$\lambda=\frac{I V-v_{1}}{v_{2}-v_{1}}$
3. Draw the point on the edge, starting from v1
4. Connect all 4 points
5. If necessary resolve ambiguities

- How can we resolve ambiguities in marching squares?

1. Mid-point decider:

Calculate isovalue in the middle by averaging all corners
Put isovalues on same side in same area
2. Asymptotic decider

- Name and explain extensions of the marching cubes algorithm!
- Isosurface reconstruction gives noisy results and contains many small objects. How can the result be cleaned up?
- What are the nodes and edges in a contour tree?

Nodes (interior): joining and/or splitting of two or more components
Nodes (leaf): creation or deletion of a component
Edge: component in the level set for all values between the values at each end of the edge

- How is the contour tree computed?

Compute join tree and split tree, then merge into contour tree

- What is the difference between the join tree and split tree construction algorithm?
Join tree: iterates over points in ascending order, creates new set if node I is a local minimum, looks ad adjacent nodes j < I , creates arc in join tree from l' to highest j' Split tree: iterates over points in descending order, creates new set if node I is a local maximum, looks ad adjacent nodes $\mathrm{j}>\mathrm{I}$, creates arc in join tree from l' to lowest j'
- Can the split tree be computed by running the join tree algorithm after simply flipping the sign of the scalar field?
- What can the contour tree be used for?

As a graph-based representation to illustrate how the topology of level sets changes with the isovalue

- Does the contour tree construction algorithm work for any dimension? Yes


## 6. Direct Volume Visualization

- Explain the Maximum Intensity Projection!

1. Raymarching (Rays from camera through data set)
2. Sampling of rays
3. Mapping (Sampled rays are interpreted)
4. Depiction of the maximum value


- What is a 2D transfer function?

Based on intensities (x-axis) and gradient (y-axis) values

- What is the difference between pre-classification and post- classification? Pre: Application of TF to all edge points in the filter range (result: RGBA quadruple); afterwards: (tri)linear interpolation of this quadruple
Post: Interpolation of the intensity values from the data (e.g., Hounsfield Units); afterwards: application of transfer function to the interpolated result
- Explain back-to-front and front-to-back compositing In volume rendering: linearly blending each layer on top of the next one from back to front or front to back, volume rendering you want front-to-back compositing
- Is ray marching unbiased?

No

- What is the free flight distance?

How far does the next photon fly in single scattering

- How can the free flight distance be used to estimate transmittance?


## 7. Elementary Methods



- Why are arrow plots mainly used in 2D only?


Arrow plots represent velocity as arrows at regular locations
In 3D there are 2 problems: ambiguity problem (it is not clearly visible which exact direction is meant/ 3D orientation because of the 2D projection) and occlusion problem (many arrows on top of each other)

- What is an effective way to visualize two scalar fields at once?
- Given a linear vector field $\mathbf{v}=(x-y, 2 x+x)$ and a seed point $\mathbf{x}_{\mathbf{0}}=(0,1)$.

Calculate the end point after integration duration $\tau=2$ analytically!

- Use a numerical integrator and see if you get the same result!
- Write the equations down for Euler, RK2 and RK4!

Euler: $x_{i+1}=x_{i}+h * u\left(x_{i}\right)$
with $\mathrm{h}=$ step size
RK2: $x_{i+1}=x_{i}+h * v\left(x_{i}+\frac{h}{2} v\left(x_{i}\right)\right)$
RK4: $x_{i+1}=x_{i}+h *\left(\frac{v_{1}}{6}+\frac{v_{2}}{3}+\frac{v_{3}}{3}+\frac{v_{4}}{6}\right)$
with
$\begin{aligned} v_{1} & =v\left(x_{i}\right) \\ v_{2} & =v\left(x_{i}+\frac{h}{2} v_{1}\right) \\ v_{3} & =v\left(x_{i}+\frac{h}{2} v_{2}\right) \\ v_{4} & =v\left(x_{i}+h * v_{3}\right)\end{aligned}$

- Explain the differences of: Euler, RK2, RK4, adaptive RK(3)

Euler has a slower convergence than RK

- RK2 is twice as expensive as Euler. Is it worth it compared to using Euler with halve the step size?
Depends on the application
- Why is streamline selection in 3D harder than in 2D? Evenly-spaced does not make sense (occlusions)
- Explain the illuminated streamline shading.


## 8. Surface-based Methods

- How many stream lines pass through one point in the domain? Only one
- How many stream surfaces pass through one point in the domain?
Only one
- How are front lines adaptively refined?

Using the Hultquist Algorithm (adapts to vertex density on front lines and flow around obstacles)
Calculate length of possible new triangle lines that connect the front lines and pick the shorter one; go to next ribbon if particles are flow orthogonal (=on the same line) again
For particles moving apart: add a new particle in the middle

- When can insertion at the seed curve become impractical?

Although insertion at the seed curve is more accurate it means that there is a single particle to seed again from the first time step. If the data is time dependent all old time steps have to read back into memory, could be huge amounts of data, it's not very efficient in terms of parallelization

- Which properties make a good stream surface?

Similar structure of shape and flow

- Why is streak surface integration more expensive than path surface integration?
Every vertex of mesh must be integrated at every time step, complete remeshing in every time step is necessary to stay within resolution constraints
- Why is the smoke surface rendering approach more efficient than the pointbased and triangle-based approach?


## 9. Topology-based Methods

- Name the parts of a topological skeleton! Explain each part!

1. Critical points

Locations where $u$ and $v$ component are equal to 0 . Classified as min/max/saddle
2. Boundary switch points

Points where some of the flow is moving out of the domain and coming back in in other areas

inbound

outbound
3. Attachment/detachment points at no-slip boundaries

There is a solid boundary where approaching particles are displaced either to the left or to the right. There is one single particle that exactly hits the boundary, this is the attachment point.


Attachment point


Detachment point
4. Separatices starting from 1-3

Streamlines created forward and backward starting from saddles, attachment and detachment points and inbound boundary switch points
Saddle: based on orientation of Eigenvalue
Inbound boundary switch points: 2 lines forward and backwards
Attachment/Detachment: 1 per point
5. Isolated closed streamlines

Global feature, acts as source or sink

- How many critical points can we have in a linear/bilinear vector field? How can we extract the critical points?
Linear: 1 solution
Bilinear: Up to 2 solutions
- How can we extract isolated closed streamlines?

For piecewise linear vector fields analyze Poincare map along a line
Grid-independent approach: create 3D vector field and seeding line, integrate and intersect stream surfaces there in both forward and backward direction, the intersection corresponds to closed streamlines

- Explain Hopf and Fold bifurcations!

Bifurcation = sudden change of the flow structure at a certain time
Fold bifurcations:
collapsing of a saddle with a source/sink/center and disappearance of both or reverse process, happens in pairs of critical points Hopf bifurcation:
repelling focus changes to attracting focus via center Attracting focus changes to repelling focus via center


- How is the feature flow field defined?

Define a vector field $f$ such that critical lines of $s$ are streamlines of $f$. The feature flow field describes the flow of certain features of a vector field $v$ over time.


Critical point tracking in v is streamline integration in $f$.
The flow vector is a constant

- What is the feature flow field in a steady flow?
- Is the feature flow field divergence-free?
- Can a streamline connect two saddles?

Yes

- Can we have critical points in time-dependent flows?


## 10. Topology-based methods II

- How many critical points can we have in a linear/trilinear vector field? Linear: 1 solution
Trilinear: up to 6 critical points
- How can we extract the critical points?

Given: Vectors ( $u, v, w$ ) at the 8 cell corners
Exit condition: all $u$ or $v$ or $w$ have the same sign
Else: recursive subdivision at ( $0.5,0.5,0.5$ ), stop if sub cells are small enough

- How many dimension(s) can a separatrix have in 3D? 2
- How can we compute saddle connectors in 3D?

A saddle connector is the curve that comes from the intersection of two separatrix surfaces

- When can we use streamline-oriented topology?

Stream lines do not carry much meaning in a time-dependent flow. There are extra steps needed first.
Better for instantaneous fields, e.g. magnetic fields

## 11. Image-based Methods

- Explain the Line Integral Convolution algorithm
General idea: global visualization technique for vector fields. Start with a random texture and smear out this texture (sampling the noise values) along the stream lines in a vector field.

For a given pixel the gray value is to be computed. The forward and backward streamlines are traced and a noise texture is sampled. All noise values sampled along the stream line are integrated (i.e. summed up) and weighted with a certain convolution weight (the further the stream line goes away from the pixel, the smaller the weight gets). The average noise value is written into the starting point.

Results: low correlation of intensity values
 between neighboring stream lines, high correlation among stream lines

- What happens if the domain is closed and LIC is computed for infinity long streamlines?
Computation gets very expensive
- How can we extend LIC to show orientation? (To show inflow, outflow)

Oriented LIC (OLIC)
Use a sparse texture (smearing of individual drops), anisotropic/asymmetric convolution kernel

- Why is LIC in 3D not used very often?

Very strong occlusion problem, you can not look into it

## Summary:

## Spot Noise

- Elongation of spots according to velocity
- Good for encoding magnitude of streamlines

Texture advection

- Advect a texture in the flow
- Reveals structures evolving over time (useful for time-dependent flow!)

LIC

- Smear gray values streamlines
- Good for encoding streamlines and shows critical points


## 12. Feature-based methods

Features: interesting structures/objects in the data (topological features, vortices, separating structures, jets, shock waves)

- What is a vortex according to Lugt?
- Describe measures for vortex extraction (vorticity, helicity, $\lambda_{\mathbf{2}}, \mathbf{Q}$, GlobusLevit, Sujudi-Haimes)

1. Region-based methods (extract the area of the vortex, i.e. scalar fields)
a) Pressure

Areas where the pressure is smaller than a certain threshold (only for 2D, invicid and steady flows)
$p \leq p_{\text {thresh }}$
b) Vorticity

Vector $=$ rotation axis, magnitude $=$ half angular speed of rotation
$|\nabla \times v| \geq w_{\text {thresh }}$
Problems: arbitrary threshold, possible false-positives in shear flow
c) Helicity
$|(\nabla \times v) \cdot v| \geq h_{\text {thresh }}$
d) Q-criterion

Positive 2nd invariant of Jacobian
$Q=\frac{1}{2}\left(\|\Omega\|^{2}-\|S\|^{2}\right)>0$
With $\Omega=$ vorticity tensor, $S=$ strain rate tensor
Vortex has more rotating motion than stretching motion

## d) Okubo-Weiß criterion

Known as the 2D version of the Q criterion
$W=-\operatorname{det}(J)<0$
e) $\lambda_{2}$-criterion
$\Omega^{2}+S^{2}=-\frac{1}{2} H(p)$
=> Problems:
Thresholds are required
All assume steady and incompressible flows
2. Line-based methods (extract the vortex core line, i.e. the center line where every thing rotates around)
a) Globus

Separation lines starting from focus saddle critical points
b) Banks-Singer Predictor-Corrector Methods

- Start from one point where we are sure it is part of the vortex core line, e.g. the local maximal of vorticity or pressure minimum.
- Take a step in vorticity direction (predictor)
- Project new location to the pressure minimum perpendicular to the voracity (corrector)
- stop if correction is too far from prediction
c) Sujudi Haimes

If I take a velocity vector sitting on the core line then its velocity direction points directly in the direction of the eigenvector with the real eigenvalue
$v-\left(v^{T} e\right) e=0$ or $v \| J v$

## d) Sahner

Construct a special vector field that allows to model ridge valley lines as integral curves

- What is Galilean Invariance and why is it important?

Find the reference frame in which the observed vector field becomes steady. Reference frame invariance: Find vortex definitions that give the same, meaningful result for every possible motion/observer.

Galilean Invariance = invariance to equal-speed translations of the reference frame
$g(x, t)=x+c_{0}+t c$ with $c_{0}=$ constant offset, $\mathrm{c}=$ constant direction
If a vortex measure is invariant to a motion with constant speed and direction, then
it is Galilean invariant. It directly translates to the ability to correctly identify vortices that perform equal-speed translations
(Example: Jacobian J, subtract feature flow field $v-\mathrm{f}$, Acceleration a $=\mathrm{J}(\mathrm{v}-\mathrm{f})$

- Which vortex measures/extractors are Galilean invariant?
$\lambda_{2}, Q$, vorticity, Okubo-Weiß (all based on J)
- Explain the reduced velocity criterion!

How can it be rephrased into parallel vectors notation?
In areas of 2 imaginary eigenvalues of the Jacobian matrix the only real eigenvalue is parallel to v . The criterion is local per cell and readily parallelized

- How can we extract parallel vectors?

Ridge lines and core lines can be explained by parallel vectors operator

- vortex core line: all locations in the domain where w 1 and w 2 are parallel
- two lines are parallel if their cross product is zero, i.e. $s=w_{1} \times w_{2}=(0,0,0)^{T}$

1. Extract and intersect isosurfaces for each component of $s$
2. Newton iterations on grid faces
3. Analytic solution on triangular faces
4. Curve following scheme => all grid dependent

- What is a streamline core / pathline core?
- Which types of Lagrangian Coherent Structures exist?

LCS: structures that govern the flow, giving the boundaries of places where particles behave the same way

1. Elliptic LCS

- coherent rotation without stretching or folding
- Lagrangian counterpart to vortex

2. Parabolic LCS

- Shearless structure
- jet cores

3. Hyperbolic LCS

- separating structures that particles cannot cross
- attracting or repelling
- approximated with FTLE
- Explain the finite-time Lyapunov exponent!
- Way for estimating hyperbolic LCS, characterizing the way of pixel separation

- Scalar field arises from integration, i.e. tracing particles for a long time
- particles would move apart and deform to a potentially complicated nonlinear deformation
- ellipse: approximation to the nonlinear deformation
- measurement for the separation: length of the biggest axis of the ellipse
- How can we compute FTLE?


## 13. Advanced Methods

- What is a tensor of rank $0,1,2$ ?

Tensor: algebraic object that describes a multilinear relationship between sets of algebraic objects related to a vector space
0 : scalar
1: vector
2: matrix

- What are the components of a stress tensor? normal stress, shear stress (3 faces, 3 options for stress => 9 components in matrix)
- What is the difference between contravariant and covariant vectors?
- Why does the ellipsoid depiction of tensors have ambiguities?

Ellipsoid shape is a 3D object placed in a 3D scene and are looked at from a 2D camera; projections might look very similar although they are different from a different camera angle
=> use superquadric instead

- Name scalar metrics for the depiction of tensors!

Goal of scalar metrics: reduce tensor data to a physically meaningful scalar field which is easy to visualize

1. Mean Diffusivity

- average of the diffusion tensor's eigenvalues (average of diagonal elements)

2. Fractional Anisotropy

- high values used as indicator for white matter
- lattice index: more noise resistant anisotropy


## 3. Linear, planar, spherical diffusion

- rotationally invariant metrics that allow to differentiate anisotropy
- metrics add up to one
- can be used to define a barycentric space of diffusion tensor shapes
- Explain the superquadric glyph for symmetric tensors!
- shape of glyph depends on cl and cp
- combines advantages of boxes and ellipsoids
- edges: strong visual cues for orientation
- ellipsoids: used when orientation is not clearly defined
- sharpness parameter can be set depending of the data's noise level
- coloring and texture for encoding of additional information
- What is tractography?

Goal: extract anatomically meaningful features
Trace in the direction of the tensor's eigenvector (integrate streamlines in the principal eigenvector field)

- What are potential problems of tractography?
- principal eigenvector is an arbitrary choice in planar and spherical regions
- noise can lead to missorting of eigenvalues

Solution: stop integration when..

- integration reaches a region with low anisotropy
- principal eigenvector is not the most collinear one
- local curvature exceeds a threshold
- What is the difference between uncertainty of visualization and visualization of uncertainty?
uncertainty of visualization:
visualization method is not accurate (e.g. humans are not precise when interpreting a grey value in an image)
visualization of uncertainty:
The data itself might be uncertain. Should be conveyed by the visualization.
- Explain what an ensemble simulation is!

Repeat the same simulation multiple times with different input conditions, compare the output of the number of simulations, discover trends and calculate probabilistic numbers. The ensemble represents N instances of the probability distribution

- Give an example for an ensemble simulation!

Weather forecasting (ensemble of 51 forecasts)

- How can we visualize scalar field ensembles?

Display probability for existence of an isocontour

- How can we visualize vector field ensembles?

Denote the probability that at location ( $\mathrm{x}, \mathrm{y}$ ) the vector has a certain value

- What is a fiber surface?

Fiber of bivariate data = intersection of two isosurfaces c, d Fibers are line structures or empty

Fiber surfaces $=$ move c and d over time, calculated by a variant of marching cubes that intersects two parameter-dependent isosurfaces

- When are fiber surfaces empty?

