

DL Introduction

Postulates for PR:

1. Availability, representative sample w of patterns if $f(x)$, field prob. \int_2
- $w = \{f_1(x), \dots, f_N(x)\} \in \mathcal{S}$
2. simple problems has features, char. membership in class \mathcal{C}_k
3. compact domain same class, separable different class
4. (complex) pattern consists of simple constituents
5. pattern $f(x)$ has certain structure, similar if features constituents differ only slightly
6. perceptions $f_j = \text{sign}(w^T x)$
- $y \in \{-1, 1\}, S = \{(x_i, y_i)\}_{i=1}^N$ set
- M set with $M \neq Y = \{\mathbf{y}_i\}_{i=1}^N$
- argmin $\|w\|_2 = -\sum y_i \cdot (\mathbf{w}^T \mathbf{x}_i)$
- $\nabla D(w) = -\sum y_i \cdot \mathbf{x}_i / \|w\|_2 + y_i \cdot \mathbf{x}_i$
- $x \in M$

Feed Forward Neural Networks

XOR problem cannot be solved, a single hidden layer already universal function approximator approximation theorem: let $p(\cdot)$ non-constant bounded measure, increasing For any $E > 0$, δ continuous, defined compact subset, $\exists N, \text{const. } \epsilon, \eta, \forall R$ and $w_i \in R^n$ with $F(x) = \sum_i v_i p(w_i x + b_i)$ with $i=1 \rightarrow |F(x) - f(x)| \leq \epsilon$ appear with just one hidden layer softmax: $y_k = \frac{\exp(x_k)}{\sum_k \exp(x_k)}$ Properties: $\sum_k y_k = 1$ Kolmogorov 2. $y_k \geq 0 \quad \forall k \in \mathbb{N}^k$ loss entropy: $H(p, q) = -\sum p_k \log(q_k)$ loss function: $L(y, p) = \sum_{k=1}^K \log(y_k)$ Softmax loss: substitute δ optimization network $L(w, x, y)$

$$E = \sum_{i,j} \delta_{ij} p_i(x_j) = \sum_{i,j} \sum_{k=1}^K \delta_{ij} (w_k x_j)$$

Targets: $w, b \in \{\ell(w, x, y)\}$

Gradient descent: $w_{\text{iter}} = w_{\text{iter}} - \eta \nabla_w L(w, x, y)$

Complex networks: $m=1$

$$L(w, x, y) = \sum_{i,j} \delta_{ij} f_j(w_i x_j)$$

$f'_j(x) = \lim_{h \rightarrow 0} \frac{f_j(x+h) - f_j(x)}{h} = \lim_{h \rightarrow 0} \frac{f_j(x + h \cdot \mathbf{x}) - f_j(x)}{h}$

chain rule: $d f_j(x) = d f_j(x) / d g(x)$

Feedback (rep): $x \rightarrow g(x)$ (controlled by y) $\rightarrow \theta \rightarrow f(g(x))$ (controlled by y) $\rightarrow \theta \rightarrow \text{grad. feed.}$ $y \nabla_w$ $\rightarrow \theta \rightarrow \text{grad. feed.}$ $y \nabla_w$

Activation functions: $f(x) = \tanh x$ $f(x) = 28(x)$ $y = \text{sign}(x)$ $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\text{grad. output} = \frac{\partial f}{\partial x}$ gradient vanishes $f'(x) = \frac{1 + \exp(-x)}{1 + \exp(-x)^2} = (1 - f(x))$ + Normalized output = gradient vanishes $\text{ReLU}: f(x) = \max(0, x)$ $f'(x) = \begin{cases} 0 & \text{else} \\ \text{const.} & \text{otherwise} \end{cases}$ + less vanishing gradient

Fully connected: $\phi = w x$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \phi} \cdot \frac{\partial \phi}{\partial w} = \frac{\partial L}{\partial \phi} \times T$$
$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial \phi} \cdot \frac{\partial \phi}{\partial x} = W^T \frac{\partial L}{\partial \phi}$$

Loss functions (Classifications, Estimates)

- a discrete variable for every input
- Regression: $\mathcal{L}(\hat{y}, y) = \|\hat{y} - y\|_2^2$
- Maximum Likelihood Estimation Given $X = x_{1:N} \in \mathbb{R}^{n \times N}$ $Y = y_{1:N} \in \mathbb{R}^{1 \times N}$ $p(y|x)$
 $\ln p(Y|x) = \ln \prod_{i=1}^N p(y_i|x_i) = \sum_{i=1}^N \ln p(y_i|x_i)$
- Logistic Regression: $\{ \mathbb{P}(Y_i=1|x_i) \}_{i=1}^N \sim \text{Beta}(\alpha, \beta)$ $\ln p(Y|x) = \ln \prod_{i=1}^N \frac{\alpha}{\alpha + \beta} x_i + \ln \prod_{i=1}^N \frac{\beta}{\alpha + \beta} (1 - x_i)$
- Linear Regression: $\{ \mathbb{P}(Y_i=y_i|x_i) \}_{i=1}^N \sim \text{Multinomial}(p_i)$ $\ln p(Y|x) = \ln \prod_{i=1}^N p_i^{y_i} = \ln \prod_{i=1}^N \frac{\exp(w^T x_i)}{\sum_{y=0}^M \exp(w^T x_i)}$

Activation functions

- Sigmoid: $f(x) = \frac{1}{1 + \exp(-x)}$ sigmoid saturates $x \gg 0$ and $x \ll 0$ Not zero - vanishes
- Mean $M = 0$ always shifted to $M > 0$ \rightarrow negative shift of success (success distribution) \rightarrow more constantly have to adapt shifts
- Tanh: $f(x) = \tanh(x)$ $f'(x) = 1 - f(x)^2$ $\tanh(x) = 2 \sigma(2x) - 1$ $\tanh(x) \rightarrow \text{minimal changes in } x \rightarrow \text{gradient vanishes}$

BIAS Variance Decomposition

$$b = h(x) - E[h], \text{Var}[h] = \mathbb{E}[(h - E[h])^2]$$
$$b = \tilde{h}(x|D), \mathbb{E}[\tilde{h}(x)] = \mathbb{E}[(h - b)^2] = (\mathbb{E}[h^2] - h^2) + \mathbb{E}[(h - \mathbb{E}[h])^2] + b^2$$

Training strategies

- Keep test data in cache
- Gradient check: centered difference numeric gradients, relative error
- Batch vs Gradient Descent: D vs t (batch size)
- Check in general capable by overfitting
- Monitor weights (e.g. dying ReLUs)
- Annealing learning rate: step decay, exponential decay, $\eta = \eta_0 e^{-kt}$ $\eta = \eta_0 / (1+kt)$
- Network in Network: 1×7 filters (convolutions), global (global) average pooling as last layer (less prone to overfitting)
- Very deep: 1×1 kernel size in conv.
- GoogleNet: Inception module

Hypoparameter optimization

Ensembling: multiple classifiers independent speed: $1-p$, kernels $\propto N/(k^2)$ $\propto 1/p$ $N \rightarrow \infty$ \rightarrow prob. majority $\propto \sum_i \mathbb{P}(Y_i|X_i) p(X_i)$

Class Imbalance: $b = \bar{M}/\bar{N}$

- Undersampling: Take subset for overrepresented data
- Oversampling: Use sample from underrepresented data (use multiple times)
- Undersampling: \rightarrow better fit can be reduced by taking a different subset, data augmentation can help by overfitting
- Adapt loss function, e.g. $L(y, \hat{y}) = -w_k \log(y_k)$, $y_k = 1$

Residual units: simple identity link

- $H(x) = F(x) - x$
- $F(x) = H(x) + x$
- $x_{i+1} = h_i(g(x_i) + h_{i+1}(x_i))$
- Original: g : identity, h : residual
- Better: g and h identity \rightarrow pre-activated
- Residual View: $x_{i+1} = x_i + h_i(x_i)$
- \rightarrow 2nd path at neuron level many different paths varying lengths

Representation View

- (a) Style (layer): direct computation
- (b) Residual Network: improved iterative estimation
- (c) Cluster network: produces new rep. each layer

Different LSTM, GRU

- LSTM: Separate hidden and cell state, controlled exposure of memory through output gate, memory content independent forget gate
- GRU: combined hidden/cell state, full exposure of memory content without control, no memory depend on cur. memory: $h_t = (1 - \zeta_t) \otimes h_{t-1} + \zeta_t \otimes \tilde{h}_t$

Sampling Strategies for RNNs

- RNN: computes prob. distribution of next state
- Greedy search: At each point, pick the most likely element (no lookahead)
- Beam search: Select b most likely elements, out of all possible sequences that have one of these elements as a prefix take b most probable ones
- Random sampling: Sample next word according to output probability distribution, temperature sampling!

Learning Architectures Goal: self-developing network structures

- Accuracy: A , FLOPs (optimization time), grid-search typically too consuming
- RNN to generate visual desc. of networks, train with reinforcement learning to max. expected accuracy
- Dropout: $\text{forget. rate} = 1 - \text{keep. rate}$ \rightarrow long term ≈ 0.9 input width l

Activation functions

- Linear: $f(x) = x$
- ReLU: $f(x) = \max(0, x)$
- Leaky ReLU: $f(x) = \max(0, x) + \epsilon \min(0, x)$
- Tanh: $f(x) = \tanh(x)$
- Sigmoid: $f(x) = \frac{1}{1 + \exp(-x)}$
- Softplus: $f(x) = \ln(1 + \exp(x))$
- Softmax: $f(x) = \frac{\exp(x_i)}{\sum_j \exp(x_j)}$
- Swish: $f(x) = x \cdot \text{sigmoid}(x)$
- ELU: $f(x) = \begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$
- PReLU: $f(x) = \max(0, \alpha_i x_i)$
- SELU: $f(x) = \text{elu}(x) + \text{scale} \cdot \text{elu}(\text{scale} \cdot x)$
- SiLU: $f(x) = \frac{x}{1 + \exp(-x)}$
- GeLU: $f(x) = x \cdot \text{erf}(\frac{x}{\sqrt{2}})$
- Gelu: $f(x) = \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right]$
- Swish-Gelu: $f(x) = x \cdot \text{erf}(x/\sqrt{2})$
- GeLU-Gelu: $f(x) = \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right] + \frac{1}{2} \left[x \cdot \text{erf}(x/\sqrt{2}) \right]$
- SiLU-SiLU: $f(x) = \frac{x}{1 + \exp(-x)} \cdot \frac{x}{1 + \exp(-x)}$
- GeLU-GeLU: $f(x) = \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right] \cdot \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right]$
- GeLU-SiLU: $f(x) = \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right] \cdot \frac{x}{1 + \exp(-x)}$
- SiLU-GeLU: $f(x) = \frac{x}{1 + \exp(-x)} \cdot \frac{1}{2} \left[x + \frac{\text{erf}(x/\sqrt{2})}{\sqrt{\pi/2}} \right]$

Networks

- ImageNet dataset (1000 classes, 144k images), vanishing/exploding gradients, 1000 layers, 1000k parameters
- Simple Recurrent networks: forward pass, backward pass, backpropagation via average pooling MLP as final classifier
- AlexNet: 8 layers, 60M parameters
- LeNet: Sequence: convolution, pooling, non-linearity, forward pass, backpropagation via average pooling
- Deep RNN: $h(t) = \text{tanh}(\mathbf{W}_h \cdot h(t-1) + \mathbf{W}_x \cdot x_t + b_h)$
- $y_t = \mathbf{o}(\mathbf{W}_y \cdot h(t) + b_y)$
- One-to-one: image class, one-to-many: caption, many-to-one: video class
- Peep RNN: $h(t) = \text{tanh}(\mathbf{W}_h \cdot h(t-1) + \mathbf{W}_x \cdot x_t + b_h)$
- $y_t = \mathbf{o}(\mathbf{W}_y \cdot h(t) + b_y)$
- One-to-many: image class, one-to-many: caption, many-to-many: video class

Backpropagation through time (BPTT)

$$L(y_t, \hat{y}_t) = \sum_{i=1}^T L(y_i, \hat{y}_i), \theta = \sum_{i=1}^T \frac{\partial L}{\partial \theta}$$
$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial h_T} \cdot \frac{\partial h_T}{\partial \theta} + \frac{\partial L}{\partial h_{T-1}} \cdot \frac{\partial h_{T-1}}{\partial \theta} + \dots + \frac{\partial L}{\partial h_1} \cdot \frac{\partial h_1}{\partial \theta}$$

Residual connections

- Standard: $g(h) = h + f(h)$
- Identity: $g(h) = h + h \circ f(h)$
- Pre-activation: $g(h) = f(h) + h$
- Layer Norm: $g(h) = \text{layernorm}(h) + f(h)$

Performance measures

- $TP + TN$ precision: $\frac{TP}{TP + FN}$
- $TP + FP$ recall: $\frac{TP}{TP + FP}$
- specificity: $\frac{TN}{TN + FP}$
- $TPV + TNV$ true positive rate: $\frac{TP}{TP + FN}$
- $F1 = 2 \cdot \frac{TPV \cdot TNV}{TPV + TNV}$

Multi-class classification

- Top-K error: True class (label not in the K classes with highest prediction score)
- Cross validation: k -fold cross validation: split data in k folds, use $k-1$ folds as training data, test on fold k , repeat k times
- Competitor classifiers: is new method better? 1 run method multiple times, use student's t-test: \rightarrow means significantly different with respect to a significance level or \rightarrow Bonferroni (correction): Probability that difference is caused by chance $<$ or \rightarrow Multiple comparisons \rightarrow For n tests $n \cdot \alpha$, choose $\alpha' = \alpha/n$ for each individual test \rightarrow Assumes independence, pessimistic \rightarrow More accurate permutation tests

ResNet vs. ResNets

- ResNets: densely connected (convolutional) networks, layer inputs: feature maps of all preceding layers, feature propagation reduces vanishing-gradient problem
- SENet: Squeeze-and-Excitation networks: Motivation: explicitly model channel inter-dependencies
- IDEA: Add bidirectional model that allows reading of channels dep. on input
- Squeeze: compress each channel into one value (global avg. pooling)
- Excitation: FC layers & sigmoid to achieve scaling vector
- Scale: Scale input feature maps

Sampling Strategies for RNNs

- RNN: computes prob. distribution of next state
- Greedy search: At each point, pick the most likely element (no lookahead)
- Beam search: Select b most likely elements, out of all possible sequences that have one of these elements as a prefix take b most probable ones
- Random sampling: Sample next word according to output probability distribution, temperature sampling!

Learning Architectures Goal: self-developing network structures

- Accuracy: A , FLOPs (optimization time), grid-search typically too consuming
- RNN to generate visual desc. of networks, train with reinforcement learning to max. expected accuracy
- Dropout: $\text{forget. rate} = 1 - \text{keep. rate}$ \rightarrow long term ≈ 0.9 input width l

